

Algebraic Geometry Mid Term

February 14, 2010

There are 6 questions of 5 points each. Please look over the entire paper before attempting to answer as some questions may be easier than others.

1. Find an example of two non-isomorphic curves in \mathbb{P}^2 which have the same Hilbert polynomial.
2. Prove that $\mathbb{A}^2 - \{(0,0)\}$ is not affine by computing its ring of regular functions.
3. Compute the Hilbert Polynomial of the d^{th} Veronese embedding of \mathbb{P}^2 .
4. Let $C = V(F)$ be an irreducible hypersurface in \mathbb{A}^2 . Show that the set of non-singular points is an affine variety and compute its co-ordinate ring.
5. Show that the Segre product of two varieties enjoys the following universal property. Suppose X and Y are two varieties and $X \times Y$ denotes their Segre product. Let $\pi_1 : X \times Y \rightarrow X$ and $\pi_2 : X \times Y \rightarrow Y$ denote the two projections. Then if Z is any variety with maps $p_1 : Z \rightarrow X$ and $p_2 : Z \rightarrow Y$ there is a unique map $\mu : Z \rightarrow X \times Y$ such that $\pi_i \circ \mu = p_i$.
6. Compute the tangent space at the origin to the twisted cubic $V(x^2 - y, x^3 - z)$. Is $(0,0,0)$ a smooth point ?